Engineering Notes

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Identification of Large Flexible Structures Mass/Stiffness and Damping from On-Orbit Experiments

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Introduction

MANY proposed space missions for the coming decades involve large space structures (LSS). The mass, damping, and stiffness characteristics will not be known accurately a priori. Active control of LSS necessitates an accurate estimate of the parameters so that control laws can be tuned on-orbit to improve the control system performance. Two approaches to the identification problem can be found in the literature. One approach is to identify the discrete or lumped parameter model of the system. The other is to identify the distributed model, i.e., the partial differential equation with the boundary conditions. This Note investigates the first approach.

Forced Vibration Method

Consider a LSS whose dynamics are governed by the following linear matrix differential equation

$$M\ddot{x}+C\dot{x}+Kx=F \tag{1}$$

where x is an $n\times 1$ configuration vector of physical displacement, M, C, and K are $n\times n$ symmetric mass, damping, and stiffness matrices, and F is an $n\times 1$ force vector. Equation (1) can be rewritten as

$$[\ddot{x}^T\dot{x}^T x^T]\begin{bmatrix} M \\ C \\ K \end{bmatrix}=F^T \tag{2}$$

where T means transpose. Consider a measurement process wherein the position, velocity, acceleration, and forces are measured at discrete instants $(t_1,t_2,\ldots,t_m)$. Also, consider the worst case in which the a priori M, C, and K matrices are unreliable and best estimates of all elements of M, C, and K need to be determined. Upon writing m equations identical to Eq. (2), one for each measurement time, the resulting m matrix equations can be collected in the following form:

$$AP=U \tag{3}$$

where A is an $m\times 3n$ matrix whose jth row contains measurements of the system response at time t_j,

$$j\text{th row of } A=[\ddot{x}^T(t_j)\dot{x}^T(t_j)x^T(t_j)] \tag{4}$$

U is an $m\times n$ matrix containing the forcing functions

$$j\text{th row of } U=[F^T(t_j)] \tag{5}$$

and P is a $3n\times n$ matrix containing the unknown mass, damping, and stiffness parameters

$$P=\begin{bmatrix} M \\ C \\ K \end{bmatrix} \tag{6}$$

To allow for measurement noise it will usually be desirable to overdetermine the system by choosing $m>3n$. This leads to the standard least squares solution[1]

$$P=LU \tag{7}$$

where the least square operator

$$L=(A^TA)^{-1}A^T \tag{8}$$

Obviously the method is straightforward, but it requires measurements (of acceleration, velocity, and displacement) that might pose practical difficulties. The least square operator can be modified to incorporate any known statistical information as

$$L=(A^TWA)^{-1}A^TW \tag{9}$$

where the weight matrix W is chosen as the inverse of the measurement error covariance matrix.

To examine the feasibility of using a smaller number of forces than the number of degrees of freedom of the system, the equations of motion are partitioned so that

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} C_{11} & C_{12} \\ C_{12}^T & C_{22} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}+\begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} \tag{10}$$

where $x_1(n_1\times 1)$ represents the degrees of freedom that are excited directly through the external forces, and $x_2(n_2\times 1)$ refers to the degrees of freedom that are excited through the coupling terms. Note that $n_1+n_2=n$. The top partition of

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Eq. (10) can be written as

$$[\ddot{x}_1^T \dot{x}_1^T x_1^T \ddot{x}_2^T \dot{x}_2^T x_2^T] \begin{bmatrix} M_{11} \\ C_{11} \\ K_{11} \\ M_{12}^T \\ C_{12}^T \\ K_{12}^T \end{bmatrix} = [F_1^T] \tag{11}$$

From the lower partition of Eq. (10) it is evident that $\ddot{x}_2$ is a linear combination of $\ddot{x}_1$, $\dot{x}_1$, x_1, $\dot{x}_2$, and x_2. Hence the A matrix is rank deficient and the least squares process fails. Therefore, it is not possible to estimate all of the system parameters unless all of the degrees of freedom of the system are directly excited by the forcing functions. This difficulty can be overcome if some of the system parameters are known. For instance, assume that the matrix M_{12} is known a priori. (It should be possible to arrange the problem so that the mass matrix is diagonal and hence $M_{12}=0$.) Then the A matrix can be formed such that

$$j\text{th row of } A = [\ddot{x}_1^T(t_j)\dot{x}_1^T(t_j)x_1^T(t_j)\dot{x}_2^T(t_j)x_2^T(t_j)] \tag{12}$$

Also,

$$j\text{th row of } U \text{ matrix} = [F_1^T(t_j) - \ddot{x}_2^T(t_j)M_{12}^T] \tag{13}$$

The P matrix is given as

$$P = \begin{bmatrix} M_{11} \\ C_{11} \\ K_{11} \\ C_{12}^T \\ K_{12}^T \end{bmatrix} \tag{14}$$

P can now be determined as before. The matrices M_{22}, C_{22}, and K_{22} are obtained in the following manner. The estimated C_{12} and K_{12} are substituted into the matrix Eq. (10) to form

$$[B] \begin{bmatrix} M_{22} \\ C_{22} \\ K_{22} \end{bmatrix} = -[V] \begin{bmatrix} M_{12} \\ C_{12} \\ K_{12} \end{bmatrix} \tag{15}$$

B and V are constructed from the measurements as

$$j\text{th row of } B = [\ddot{x}_2^T(t_j)\dot{x}_2^T(t_j)x_2^T(t_j)] \tag{16}$$

$$j\text{th row of } V = [\ddot{x}_1^T(t_j)\dot{x}_1^T(t_j)x_1^T(t_j)] \tag{17}$$

The linear system of Eq. (15) can be solved through least squares to get M_{22}, C_{22}, and K_{22}.

Free Vibration Method

The free-response problem based on the above formulation degenerates to the method discussed in Ref. 2.

Consider the free vibration problem for an undamped system

$$M\ddot{x} + Kx = 0 \tag{18}$$

Since M is positive definite M^{-1} exists, therefore

$$\ddot{x} + M^{-1}Kx = 0 \tag{19}$$

Equation (19) can be rewritten as

$$x^T[M^{-1}K] = -\ddot{x}^T \tag{20}$$

Let $D=M^{-1}K$. D is now determined by the least squares method. The A matrix in this case is formed such that

$$j\text{th row of } A = x^T(t_j) \tag{21}$$

Also

$$j\text{th row of } U = -\ddot{x}^T(t_j) \tag{22}$$

Notice that all modes should participate in the free response. Otherwise, the A matrix becomes rank deficient. Also note that the system eigenvalues and eigenvectors can be obtained from D. Even for low-dimensioned systems, failure to excite a particular mode means that the corresponding eigenvalues and eigenvectors cannot be obtained. Broadband random initial excitation at several stations is recommended in Ref. 3. M and K cannot be estimated explicitly without a priori knowledge of the mass matrix.

The free-vibration method can be generalized. Consider a dynamic system expressed as[4]

$$M\ddot{x}(t) + (G+C)\dot{x}(t) + (K+H)x(t) = 0 \tag{23}$$

where M, C, and K are as defined earlier, G is a gyroscopic matrix, and H is a circulatory matrix. G and H are skew symmetric. Premultiplying Eq. (23) by M^{-1}

$$\ddot{x} + M^{-1}(G+C)\dot{x} + M^{-1}(K+H)x = 0 \tag{24}$$

Equation (24) can be written as

$$[\dot{x}^T x^T] \begin{bmatrix} (G+C)^T M^{-T} \\ (K+H)^T M^{-T} \end{bmatrix} = [-\ddot{x}^T] \tag{25}$$

Thus, $M^{-1}(G+C)$ and $M^{-1}(K+H)$ can be determined. The eigenvalue problem is solved from a single matrix D given as[4]

$$D = \begin{bmatrix} M^{-1}(G+C) & -M^{-1}(K+H) \\ I & 0 \end{bmatrix}$$

The free-response data can therefore be used to determine the eigenvalues and eigenvectors of a general dynamic system.

Conclusions

Two methods are presented to identify the vibration parameters of the system. Both methods require the measurement of acceleration, velocity, and displacement at all degrees of freedom. Application of these methods to a few examples indicates that a bang-bang type of excitation, whose frequency content is wide enough to excite all the states, is a suitable excitation signal. Further research is needed to minimize the number of measurements and to optimize the excitation.

References

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